Low Energy Transfers to the Moon and Beyond:
Exploiting Resonance Transitions

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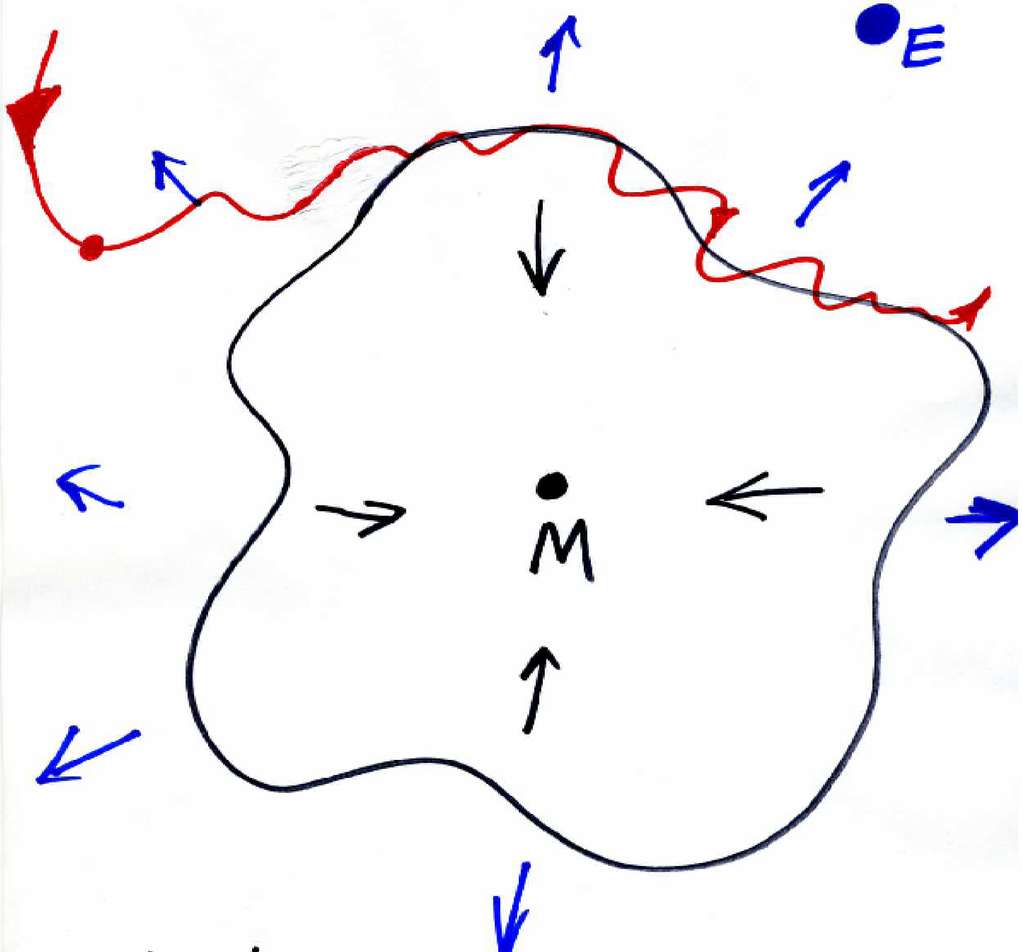
Background

- *Capture Problem*
- Earth -> Moon (LEO -> LLO)
- Prior to 1985: Solution?
- Hohmann Transfer:
- ***Fast*** (3days), ***Fuel Hog*** (Need to slow down! 1 km/s to be captured – large maneuver) ***Risky***
- Used in Apollo, etc.

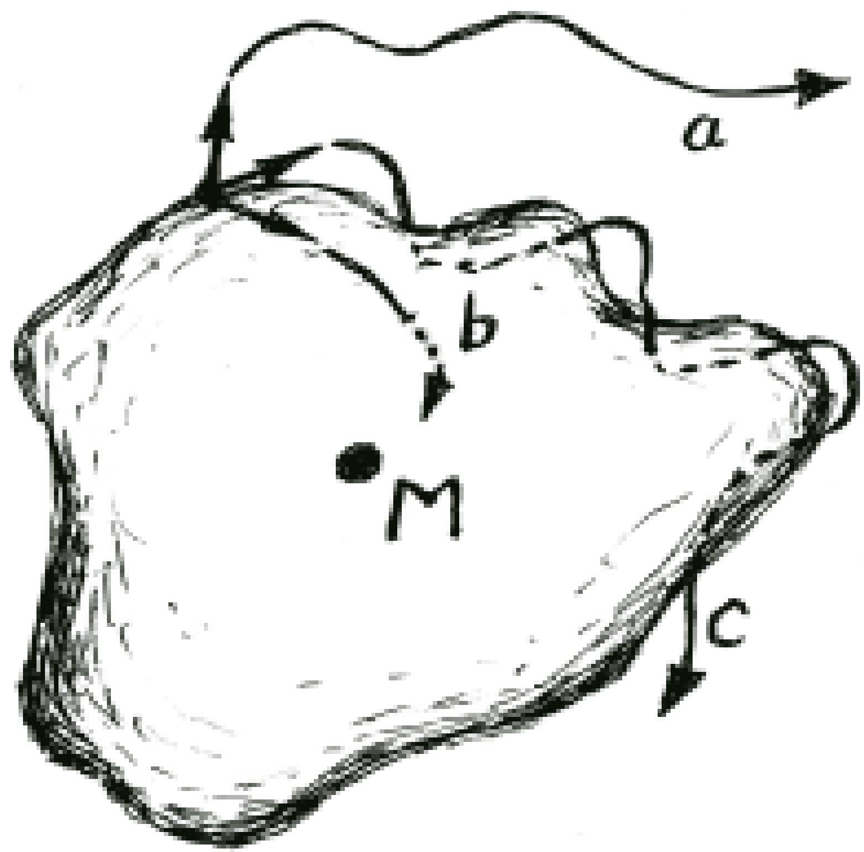
- HT – Two-Body, 1920’s technology
- Automatic capture possible? (*Ballistic capture* – i.e. no maneuvers)
- Prior to 1985, didn’t exist (Conjectured by C. Conley, 1968. Theoretically possible by ideas of Alekseev, Sitnikov, 1960s)
- Skeptical response on possibility (100 years to Moon?)
- If possible – *chaotic*
- Weak Stability Boundary (EB, 1986)

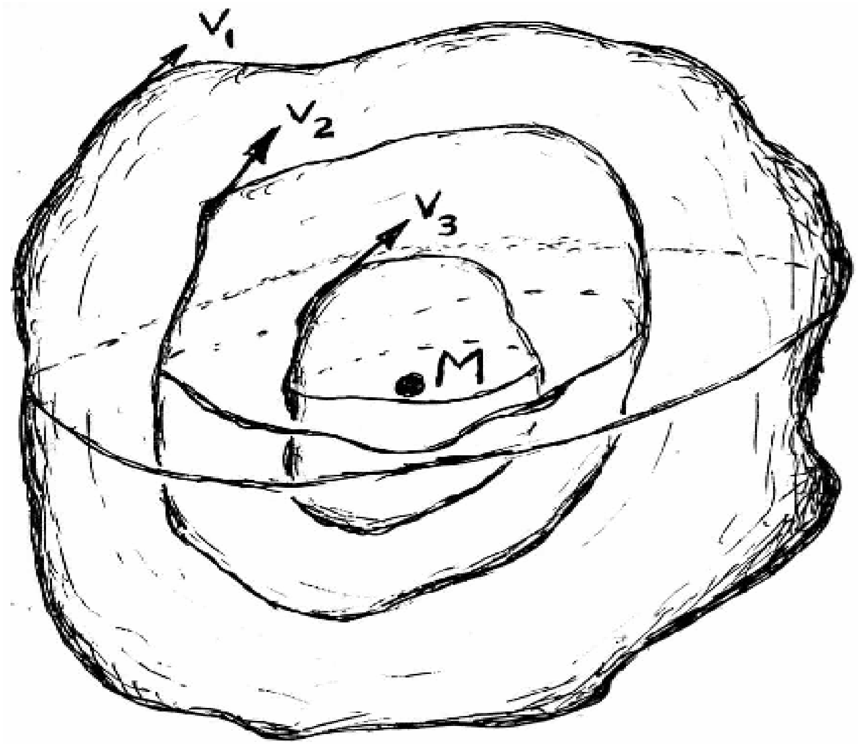
- WSB – *generalization of Lagrange points*
- Forces(GM, GE, CF) balance while spacecraft moving wrt Earth-Moon (L-points – forces balance for spacecraft fixed wrt Earth-Moon)
- Get a multi-dimensional region about Moon. Can map out on computer via algorithm.
- While in WSB, motion unstable, chaotic, but capture wrt Moon obtained - weak.



L-pts. V wrt $M = 0$ & $F = 0$

WSB V wrt $M \approx 0$ & $F \approx$


a
b
M
c



V1
V2
V3
M

- In 1986 found ballistic capture transfer to Moon for first time – 2 yr route. (Taken as oddity at that time) (*low energy* since no DV for capture)
- Lunar Get Away Special(LGAS) – first use of chaos in space travel for capture. (In Spain, Llibre, Simo used chaos to control halo orbits in 1986)
- Shorter time ballistic capture transfer not found-left JPL in 1990. While leaving, luckily found a short one(3 months) to rescue a Japanese lunar spacecraft *Hiten*.

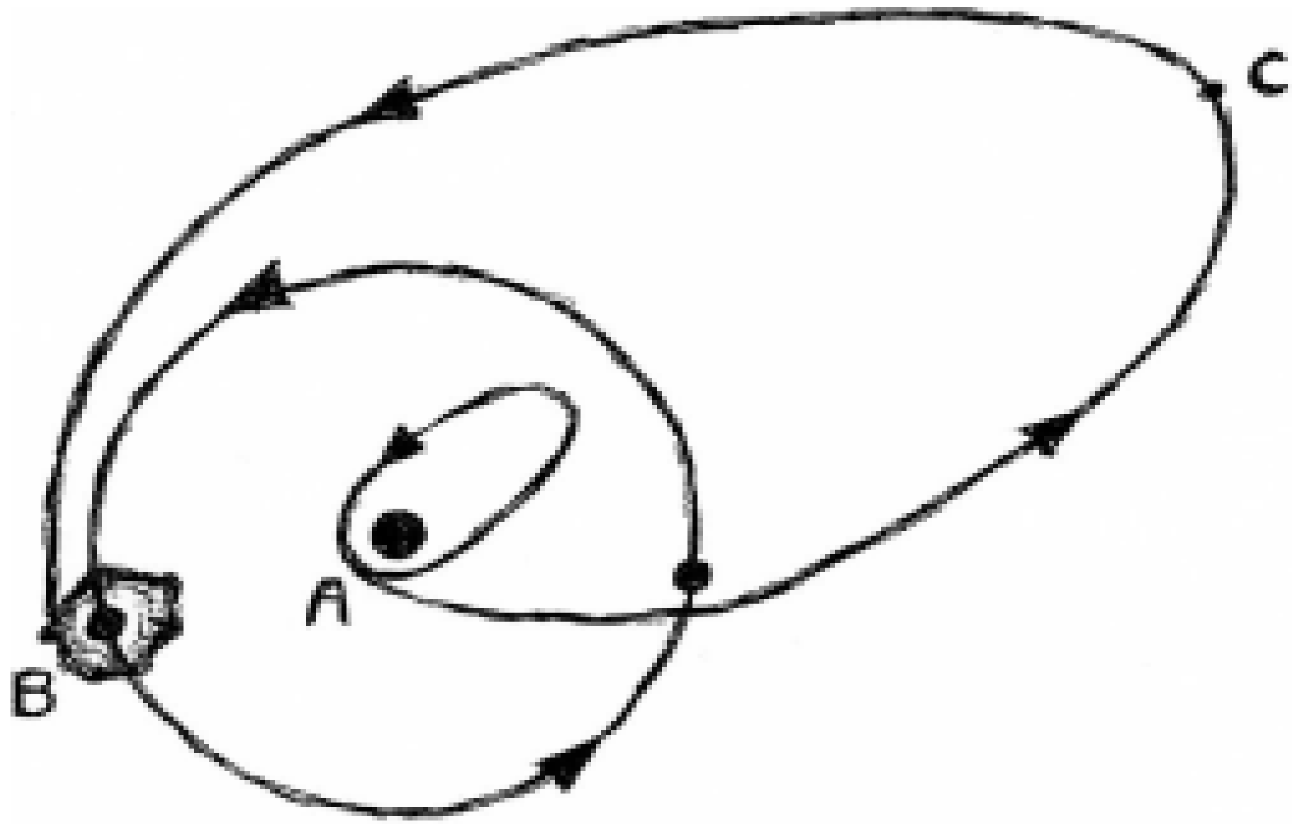
- Remarked that a ballistic capture transfer can be thought of as sneaking up on a gravitational ridge and balancing on it -

- Analogous to a surfer catching a wave.

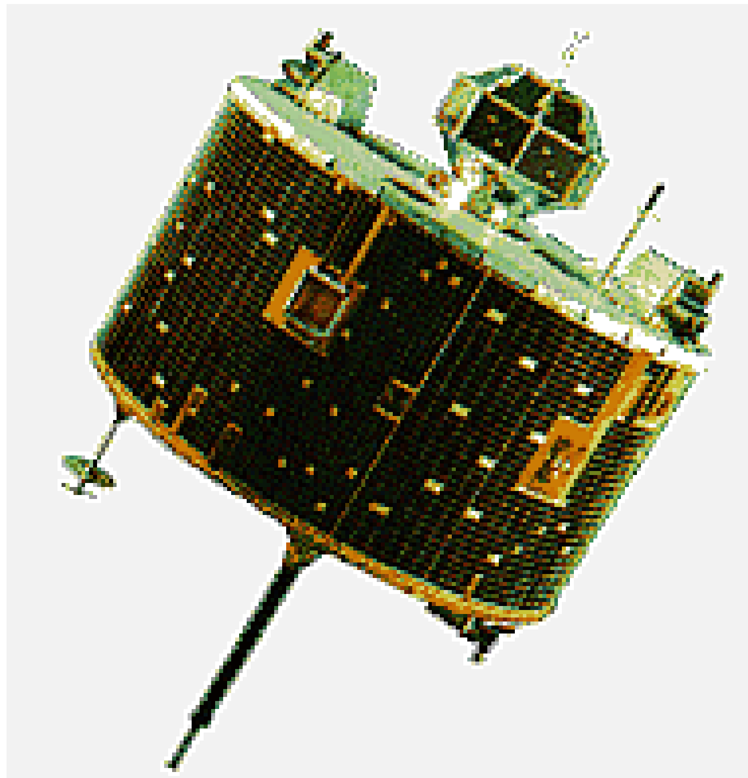
- *Surfing the gravitational chaos*

Key Observations

- Four-Body Problem
- Interlink weak stability boundaries:
- E-S $\rightarrow$ M-E
- Solves Conley Conjecture


C
A
B

- *Hiten* reached Moon on a new transfer on October 2, 1991 – first operational demo of a chaotic transfer, proving methodology

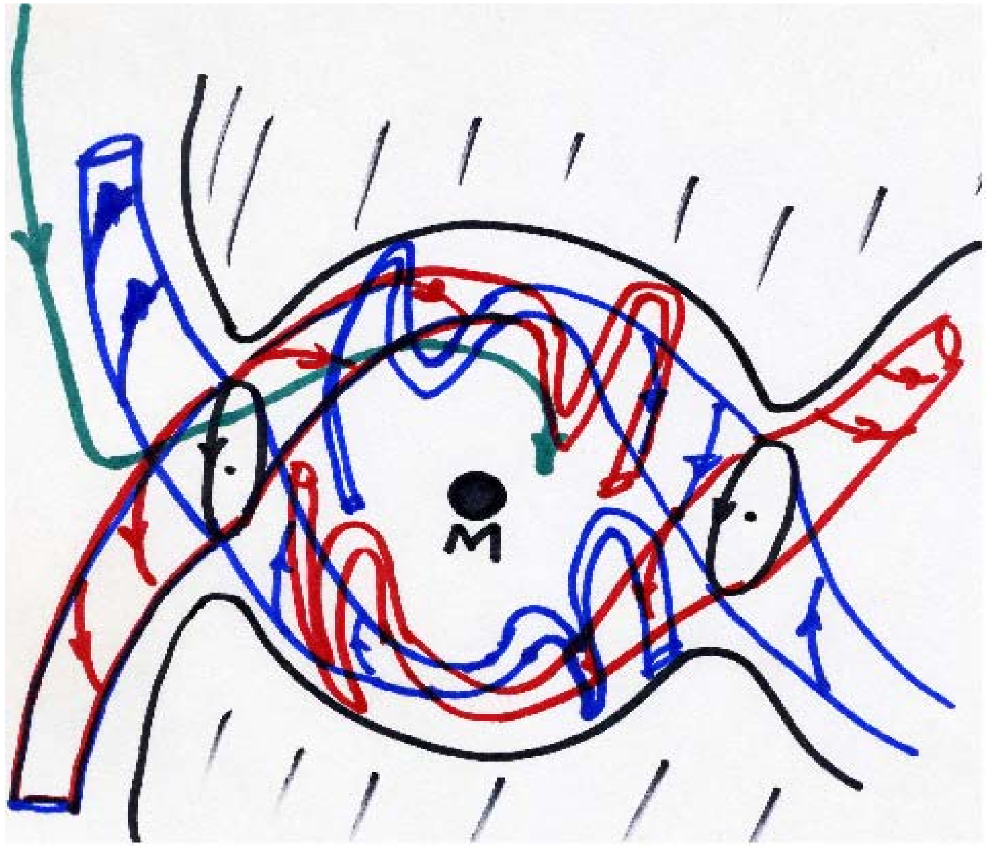


Global Structure Indicated

- Leaving Earth itself on tube and also arriving at Moon. WSB seemed to consist of invariant tubes - 1994 (EB)*,**
- Rigorous proof of tube structure leading to chaos on and near the WSB – 2004(EB) (higher energies needed – not general enough)

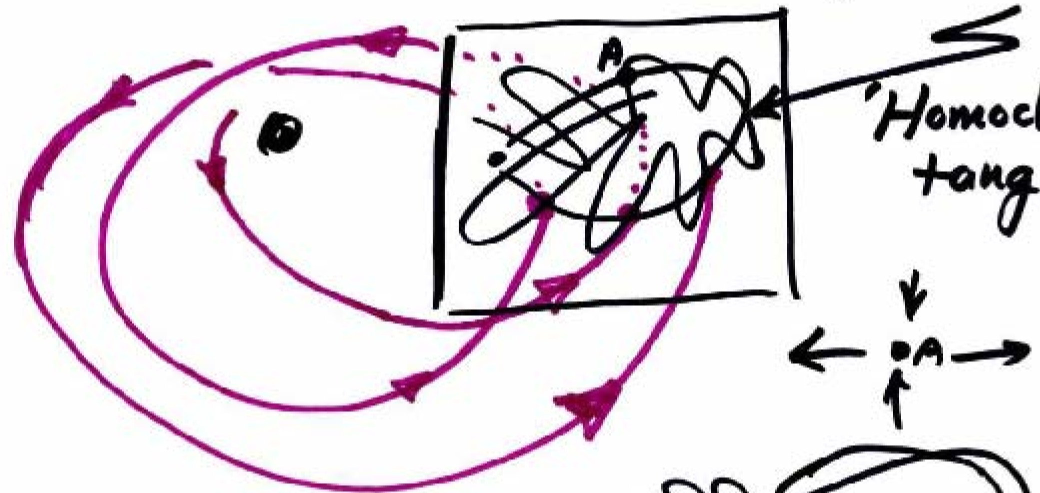
*Marsden, Lo, Ross, Koon further studied methodology outlined in 94 paper in latter 90s. Applied ideas of Llibre, Simo, EB to *Genesis* mission in 1998

**General network of tubes between bodies sometimes referred to as 'interplanetary super highway' in popular literature


M

Based on results for Sitnikov problem – suspects in 1986–90, WSB had 'hyperbolic structure'

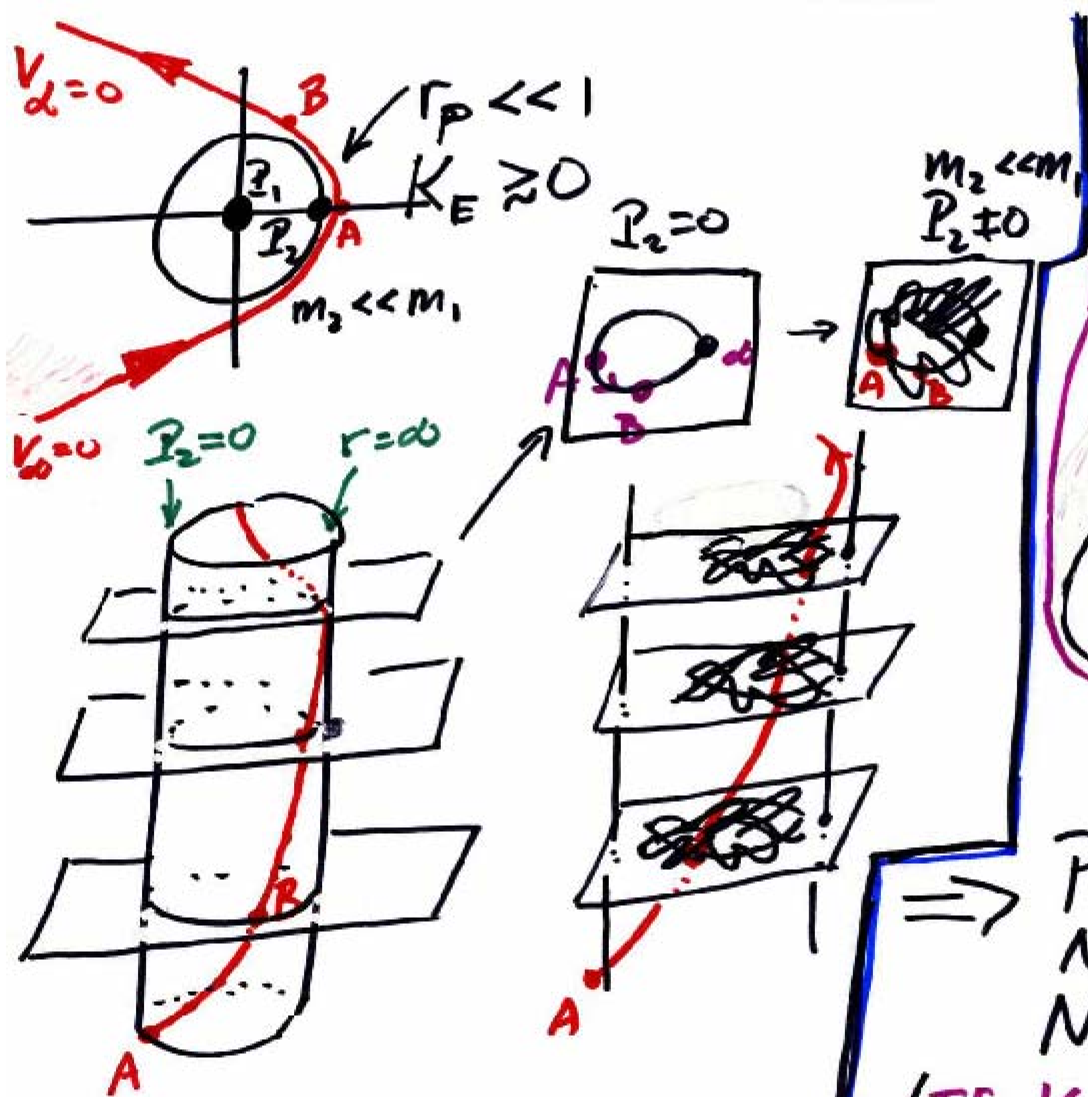
(Smale-Birkhoff Thm. $\Leftrightarrow$ homoclinic tangle $\Rightarrow$ chaos)

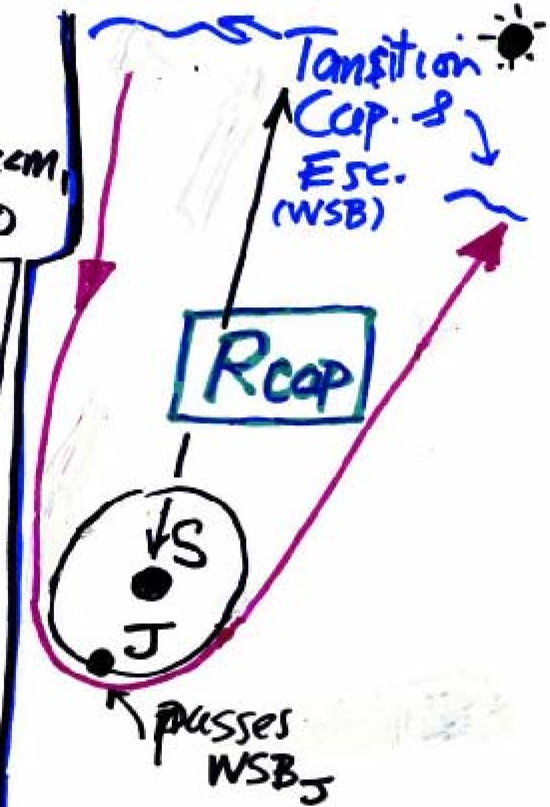


2004 – "Cap. Dyn. & Chaotic Motions in Cel. Mech." PU?

Showed (Proved) in three-body problem

Many applications





$\Longrightarrow$ Parabolic &
Near-Parabolic
Motion chaotic
(If K_E(Jupiter) $\gtrsim 0$)

Note

- 1986 original two-year ballistic capture(WSB) transfer, used by ESA's **SMART-1** in 2004
- WSB transfer planned for ESA *BepiColombo* Mercury mission
- Three month WSB transfer saves **25%** in DV to place payloads onto the Moon or into lunar orbit – ***can double payload***.
- Ideal choice for a *lunar base construction*

WSB?

- General nature of WSB elusive
- Work in 90s (EB, 90)suggested gave rise to resonant motion wrt Earth in resonance with the Moon (or wrt Sun in resonace wrt Jupiter – hopping comets Oterma, etc – EB, B. Marsden; AJ 1997)
- General nature recently uncovered in AISR project

- **Idea** – WSB (or weak capture) *hub* for resonance transitions
- Start on WSB=> Trajectory in forward or backward time on ***m:n*** resonance orbit (spacecraft **m** cycles, Moon **n** cycles)
- Or comet m cycles, Jupiter n : example Gehrels 3 3:2 ->weak capture-> 2:3
- ***Resonance hop*** (resonance transition) *

*EB/BMarsden 97 paper studied further by MMarsden,Lo,Ross in 2000, but energies too restricted, and no insight into WSB

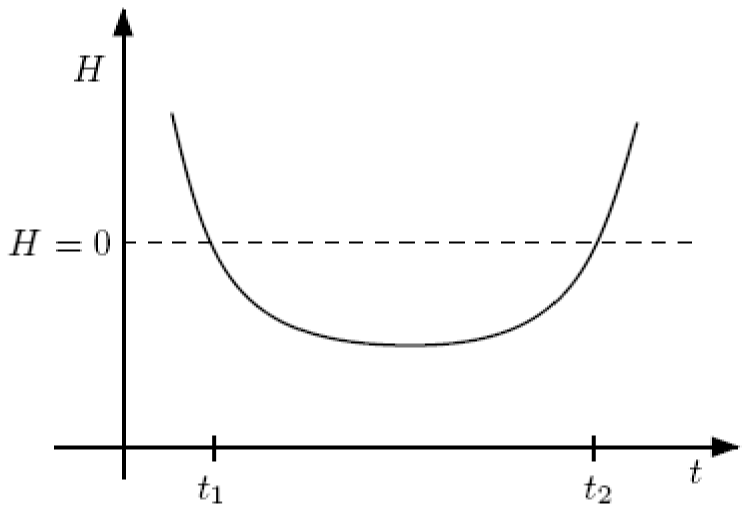
- Model: Planar circular restricted three-body problem: P0, P1(1-mu), P2(mu)
- Assume mu = .01215… (Earth, Moon)
- Rotating frame, cm at origin
- x,y,dx/dt,dy/dt, on J=C
- Earth: x=-mu, Moon, x-1-mu

C: 3.024…, 3.184…, 3.200…

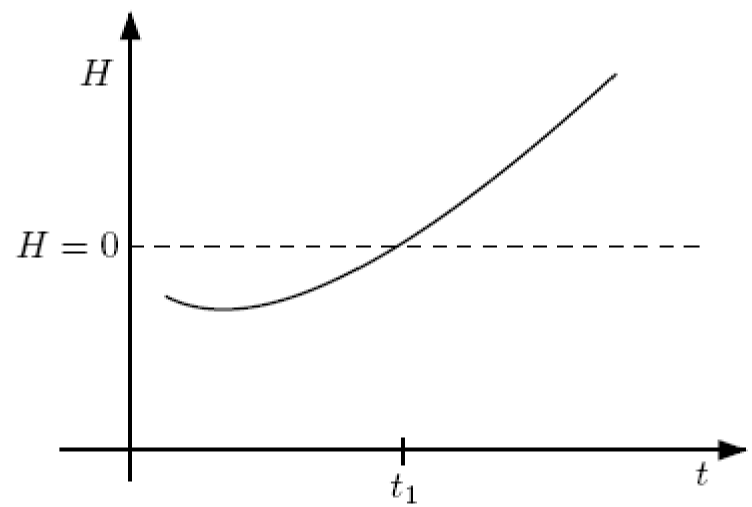
L3 , L1 , L2

As C decreases. P0's motion becomes more energetic

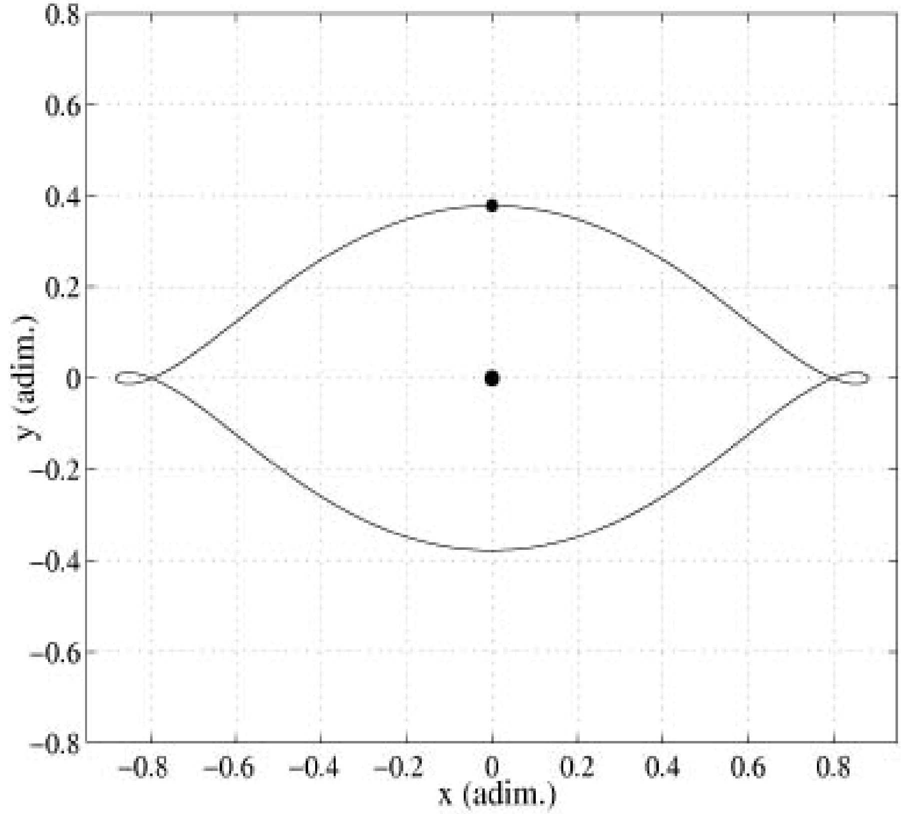
Weak Capture



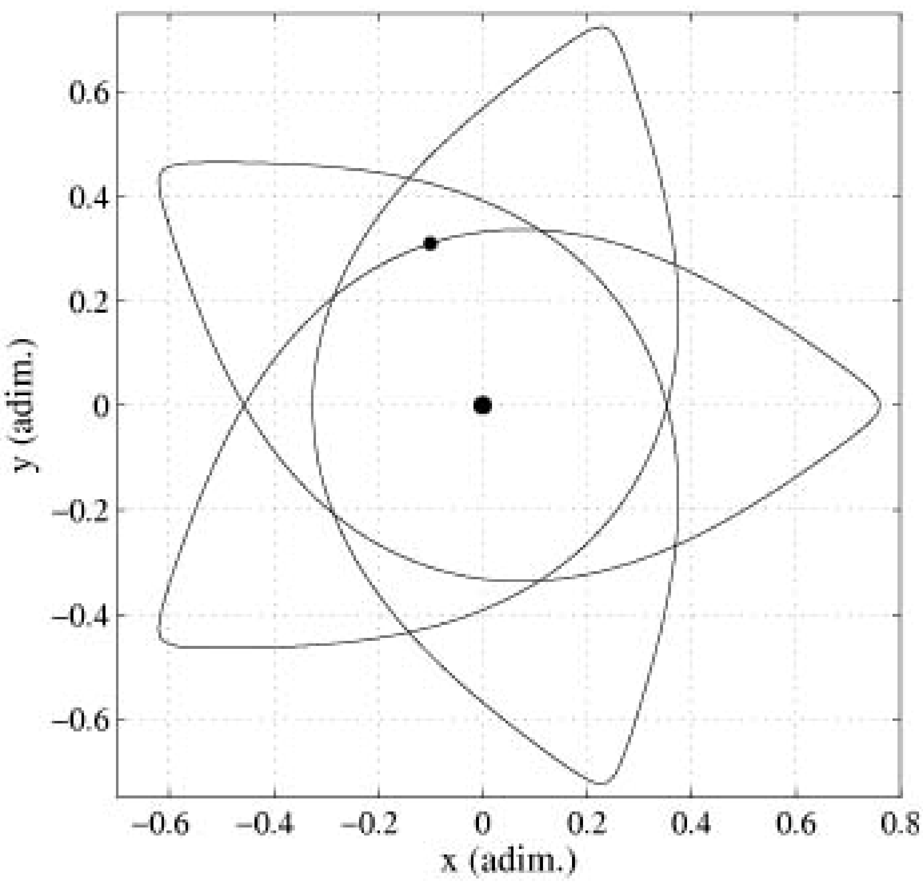
(a) Temporary ballistic capture.



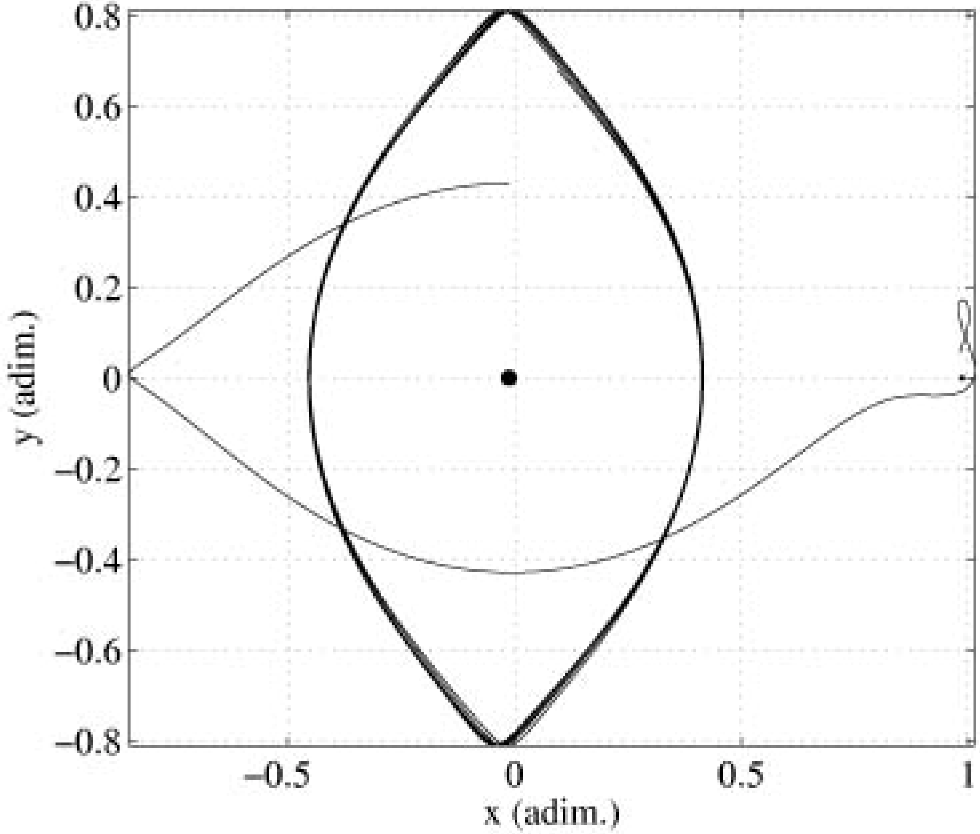
(b) Ballistic ejection.

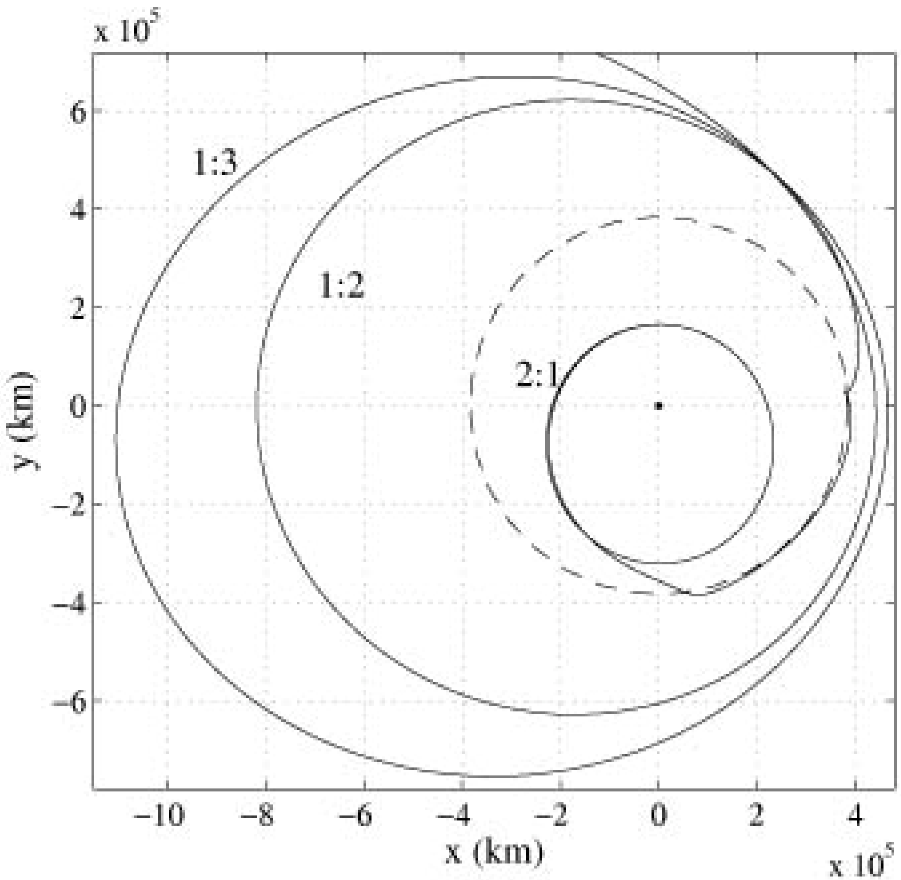


2:1

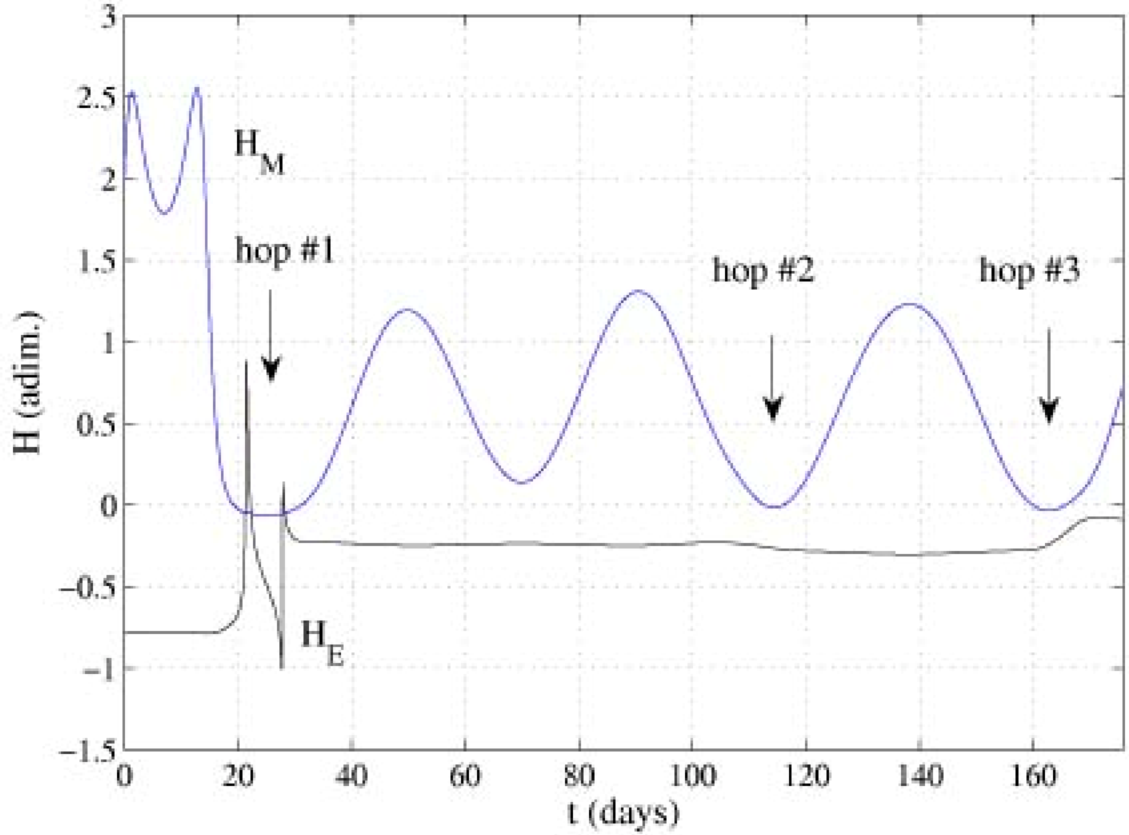


5:2


0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
y (adim.)
−0.5
0
0.5
1
x (adim.)



Resonance hops



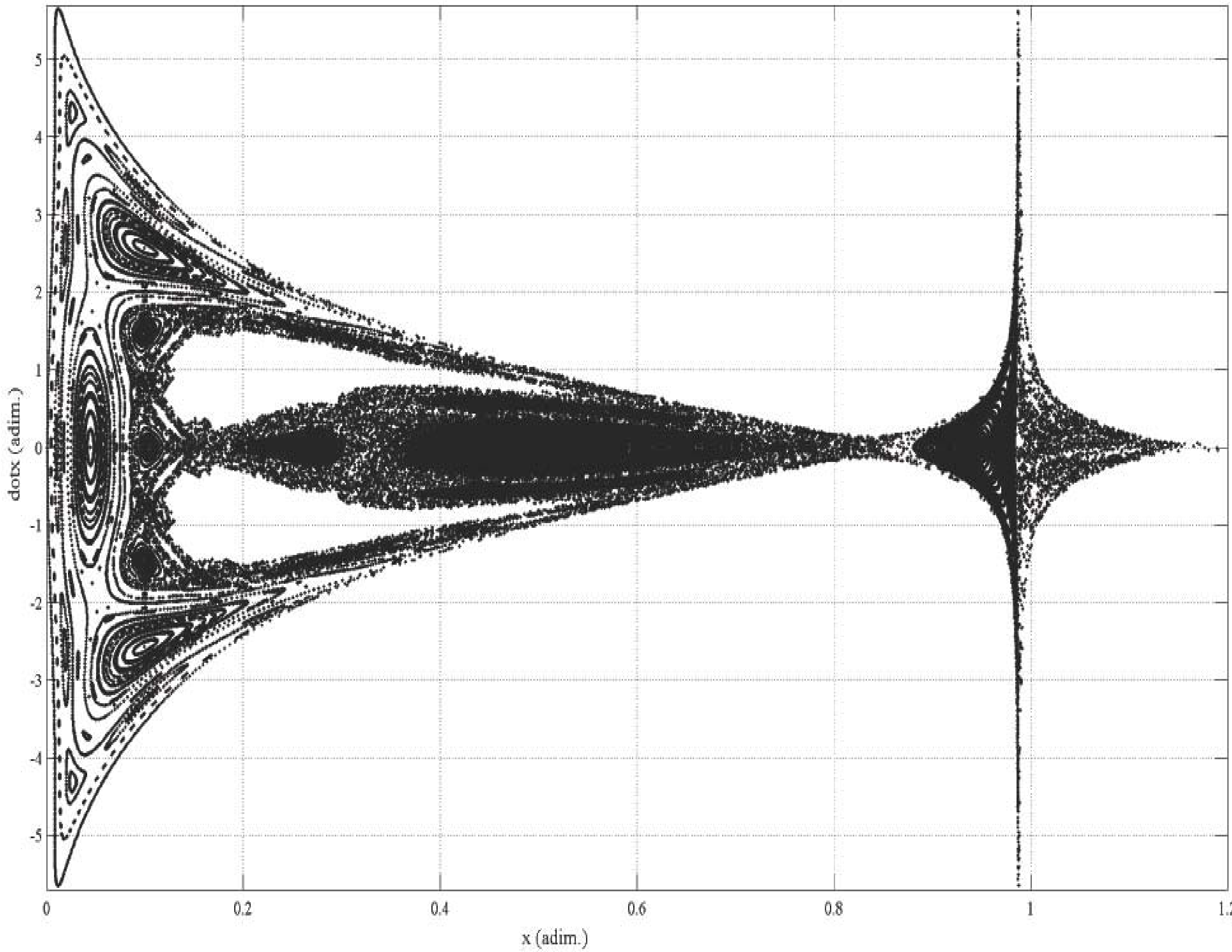
H (adim.)
t (days)
H_M
H_E
hop #1
hop #2
hop #3
3
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
0
20
40
60
80
100
120
140
160

Key: Special Poincare Sections

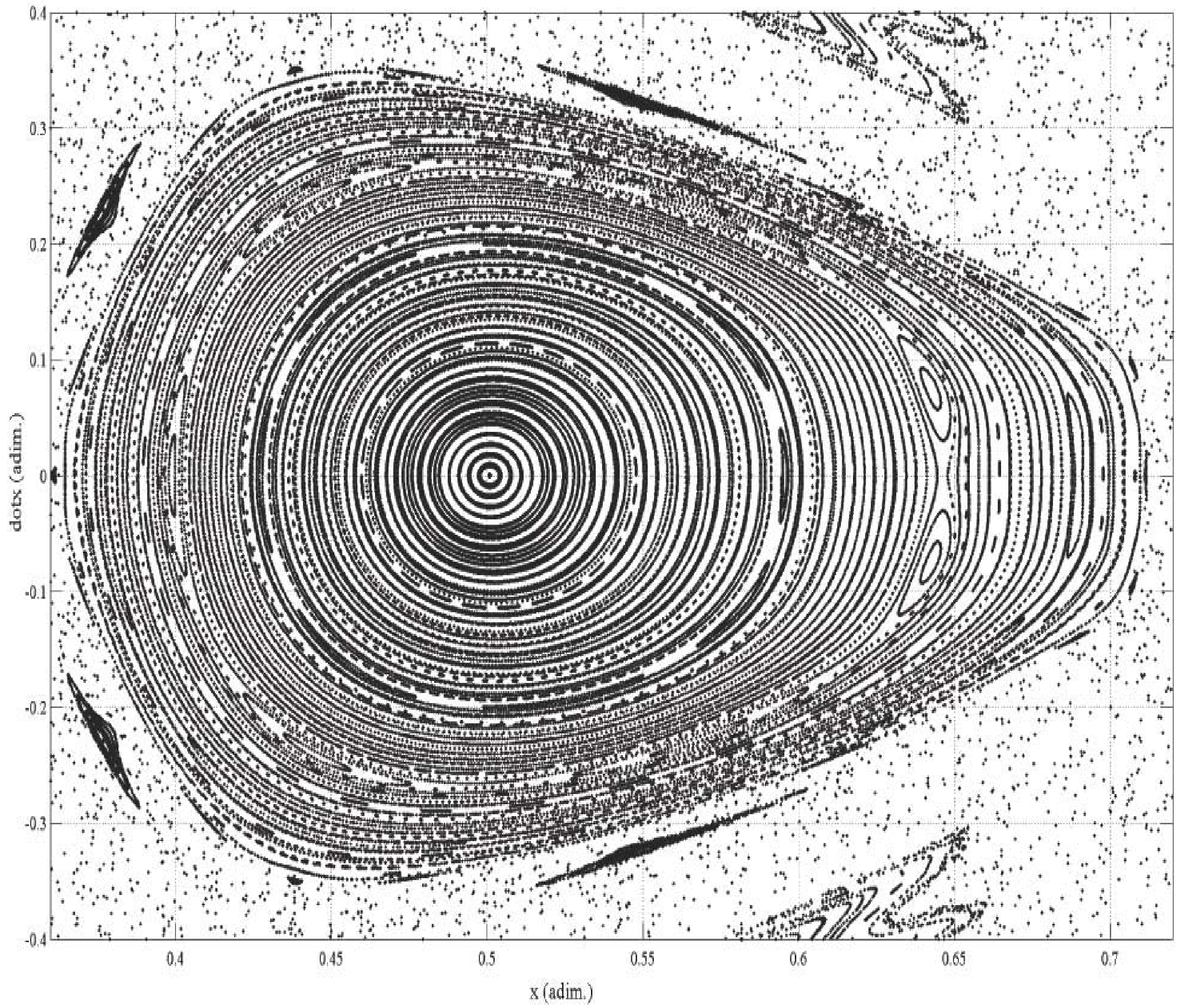
Sections defined from using 2:1 ICs
C(a,e,omega,theta) => *C(e)*

For a fixed energy J = C(e), sections are generated along the x-axis between the Earth and Moon, via 300 x values, with 1000 iterations each. (e varies between 0,1)

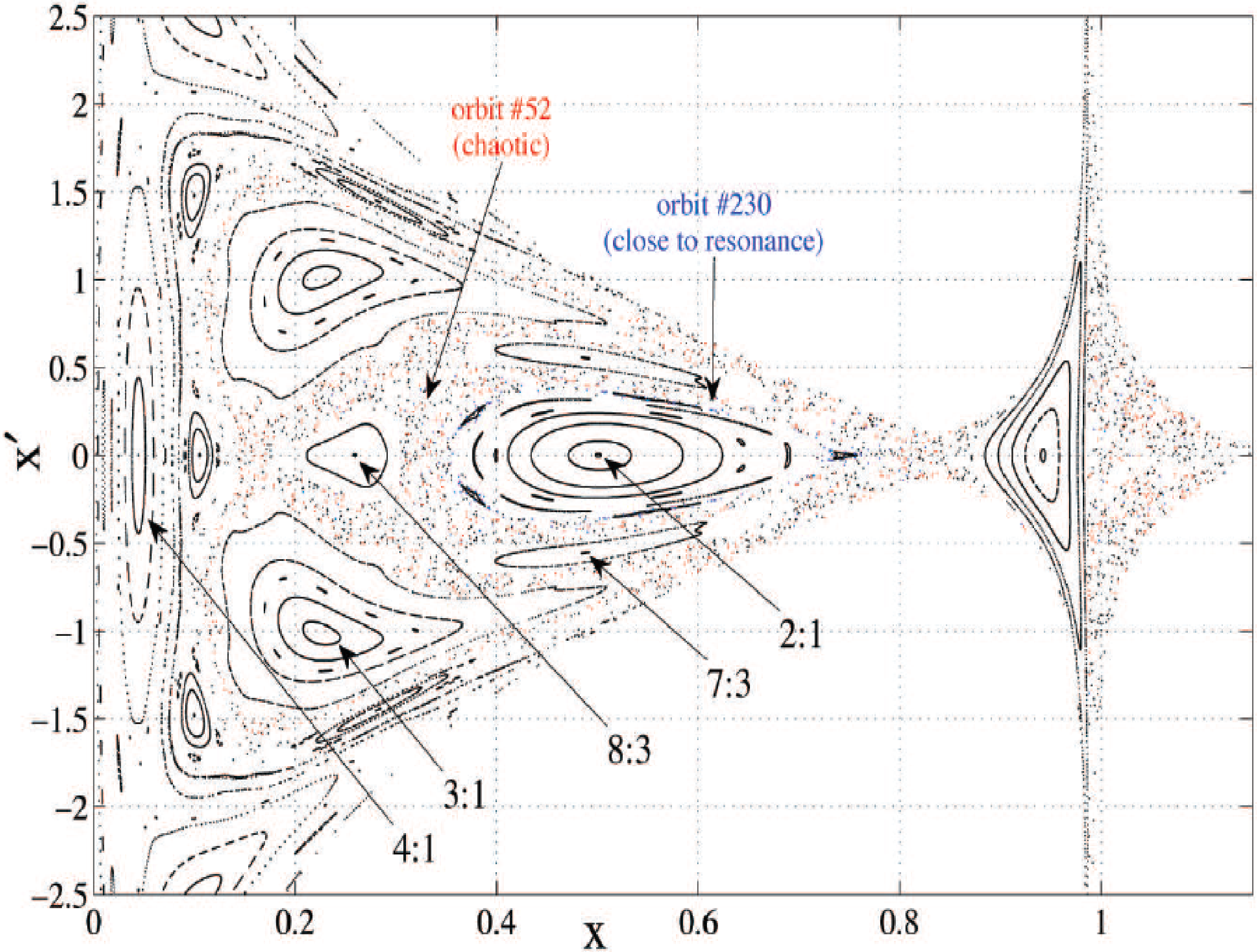
Surfaces viewed in different coordinates:
(x,dx/dt), (a,omega)



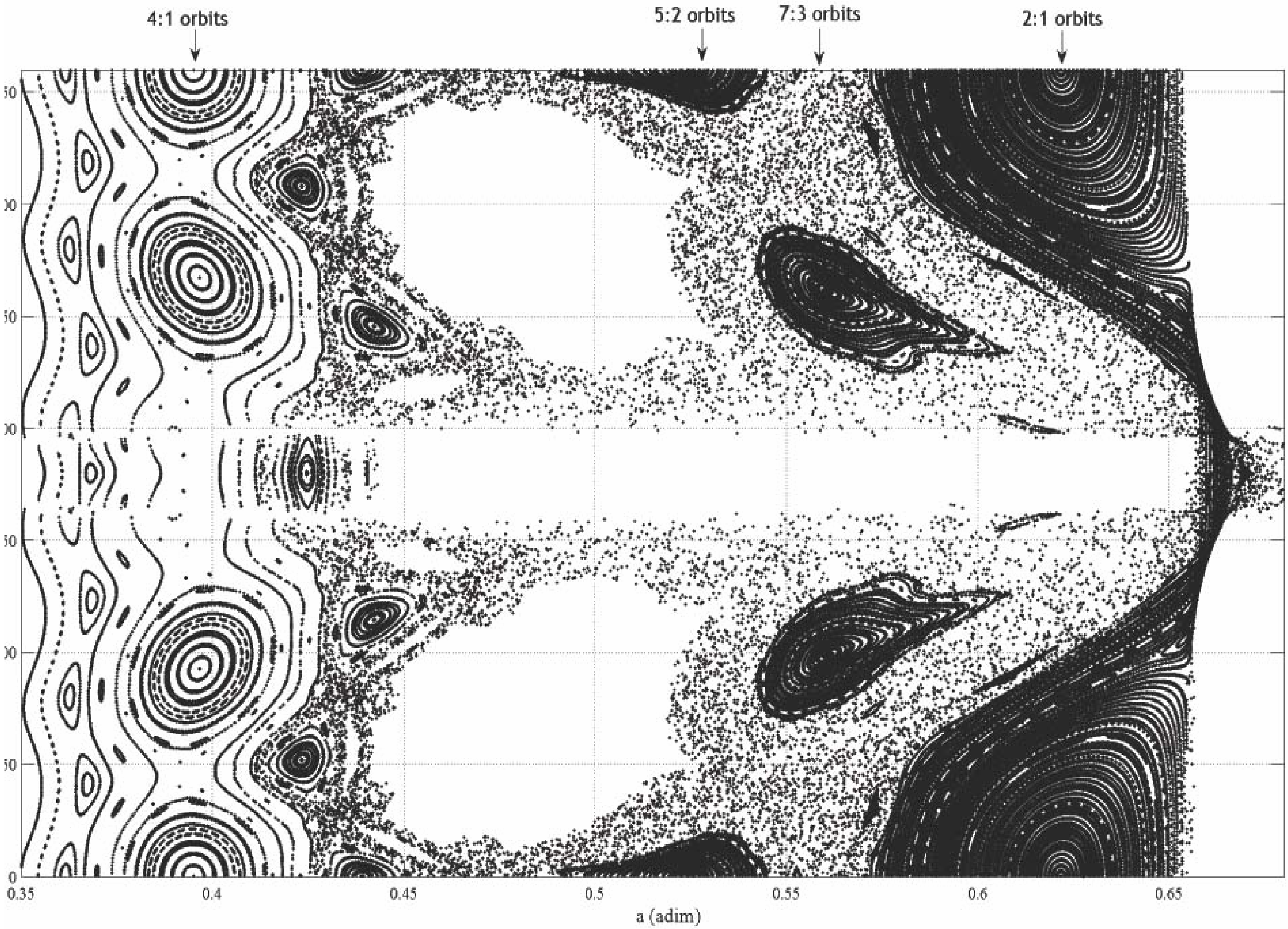
C= 3.1817683176, e=0



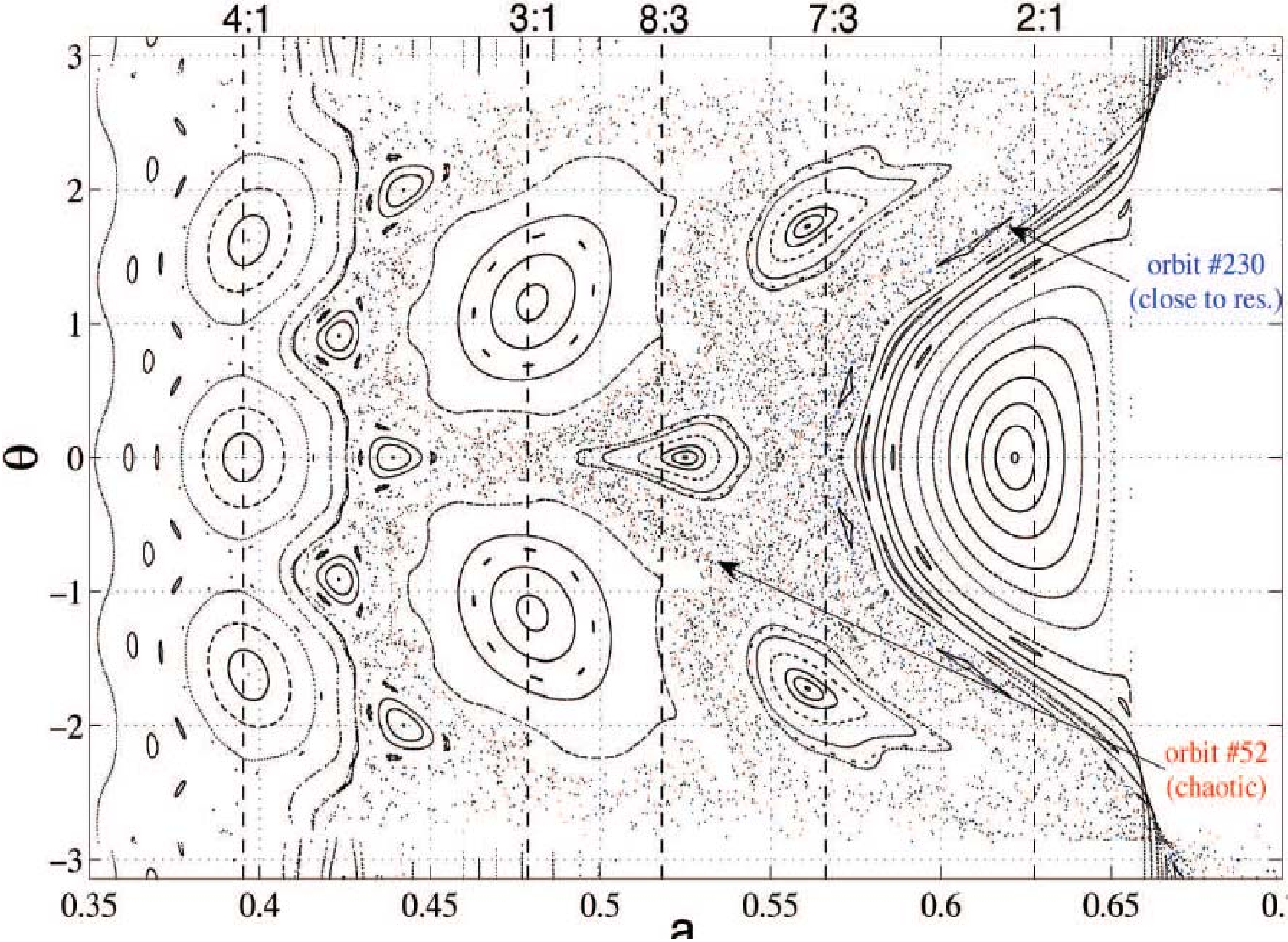
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
dotx (adim.)
0.4
0.45
0.5
0.55
0.6
0.65
0.7
x (adim.)



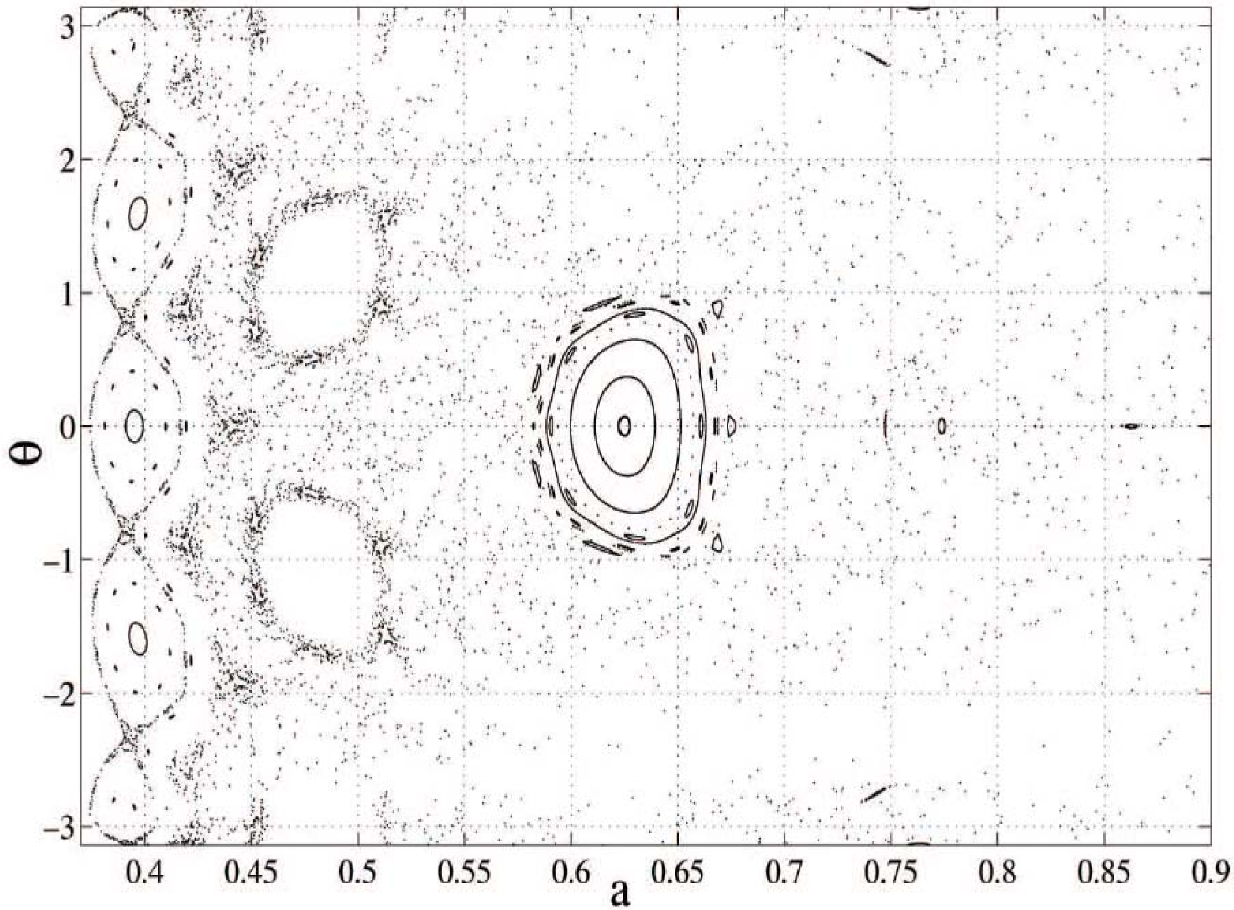
orbit #52
(chaotic)
orbit #230
(close to resonance)
2:1
7:3
8:3
3:1
4:1
x'
x
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
0
0.2
0.4
0.6
0.8
1



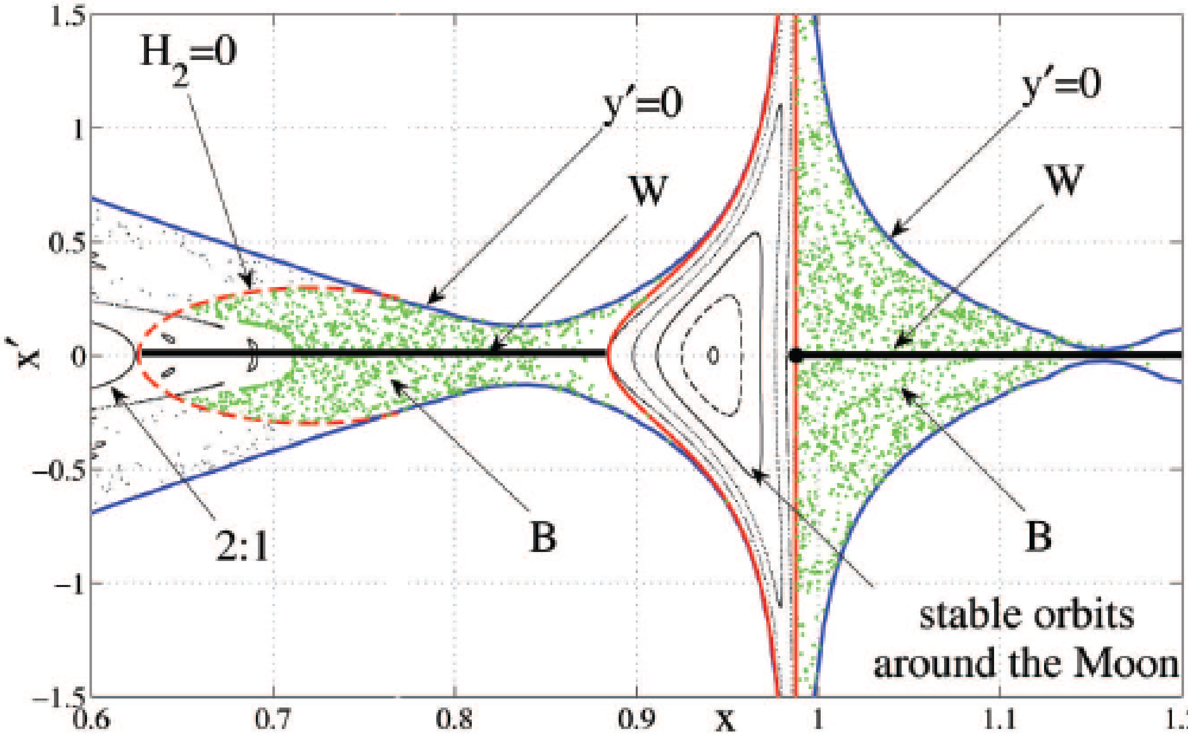
C= 3.1817683176

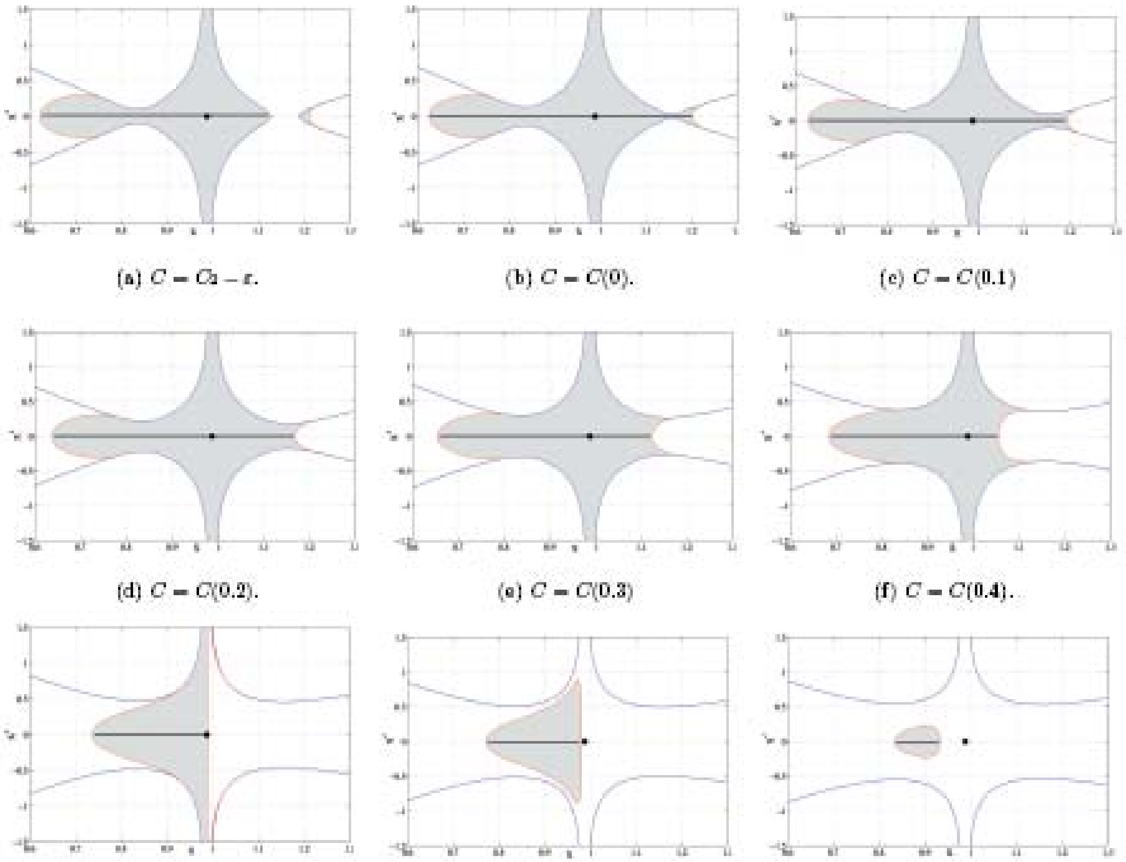


4:1
3:1
8:3
7:3
2:1
orbit #230
(close to res.)
orbit #52
(chaotic)
θ
a
3
2
1
0
−1
−2
−3
0.35
0.4
0.45
0.5
0.55
0.6
0.65



C=2.8698501942 e=.6


H2=0
y′=0
W
y′=0
W
x′
2:1
B
B
stable orbits
around the Moon
x
1.5
1
0.5
0
−0.5
−1
−1.5
0.6
0.7
0.8
0.9
1
1.1
1.2



(a) $C = C_2 - \varepsilon$.

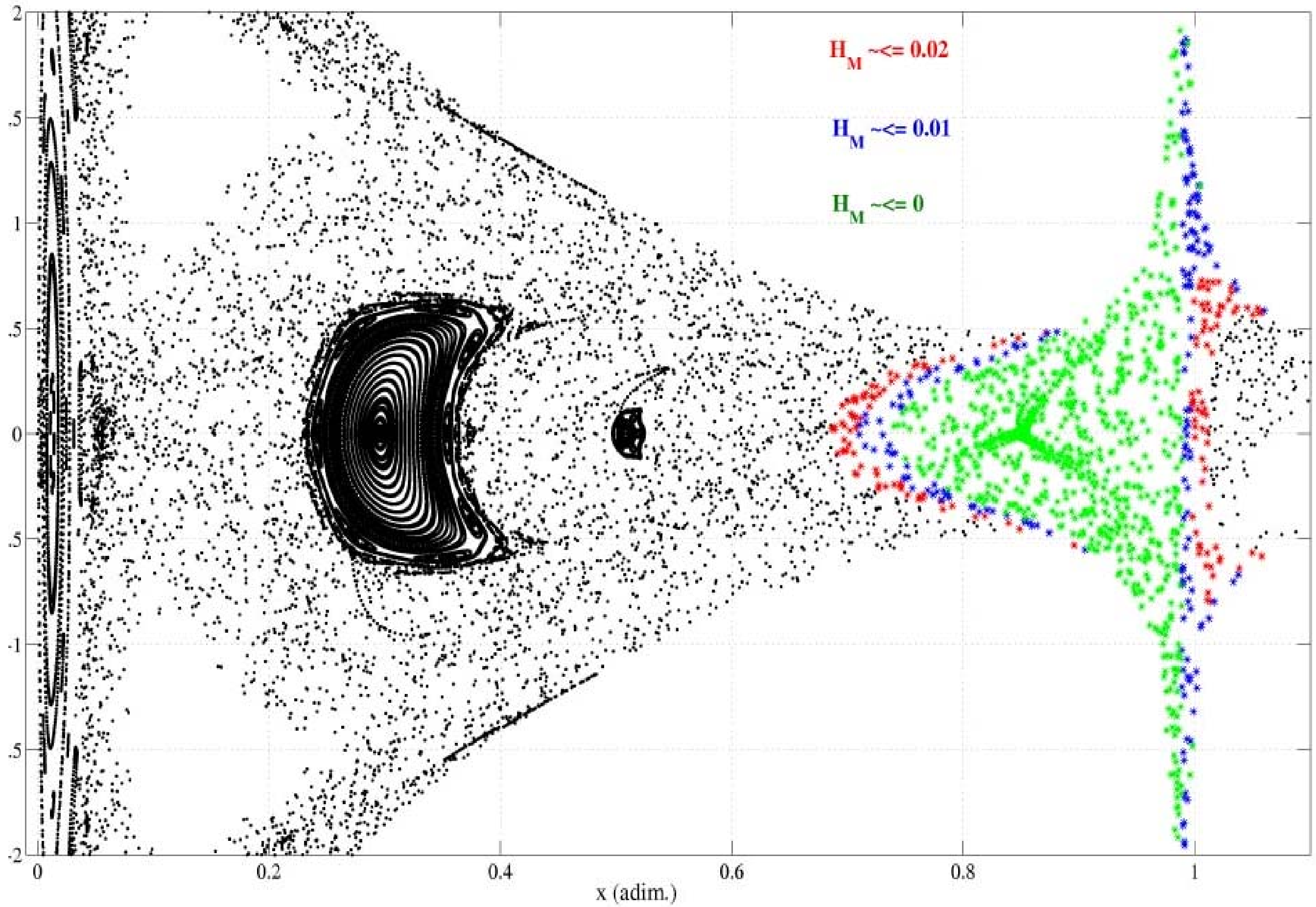
(b) $C = C(0)$.

(c) $C = C(0.1)$

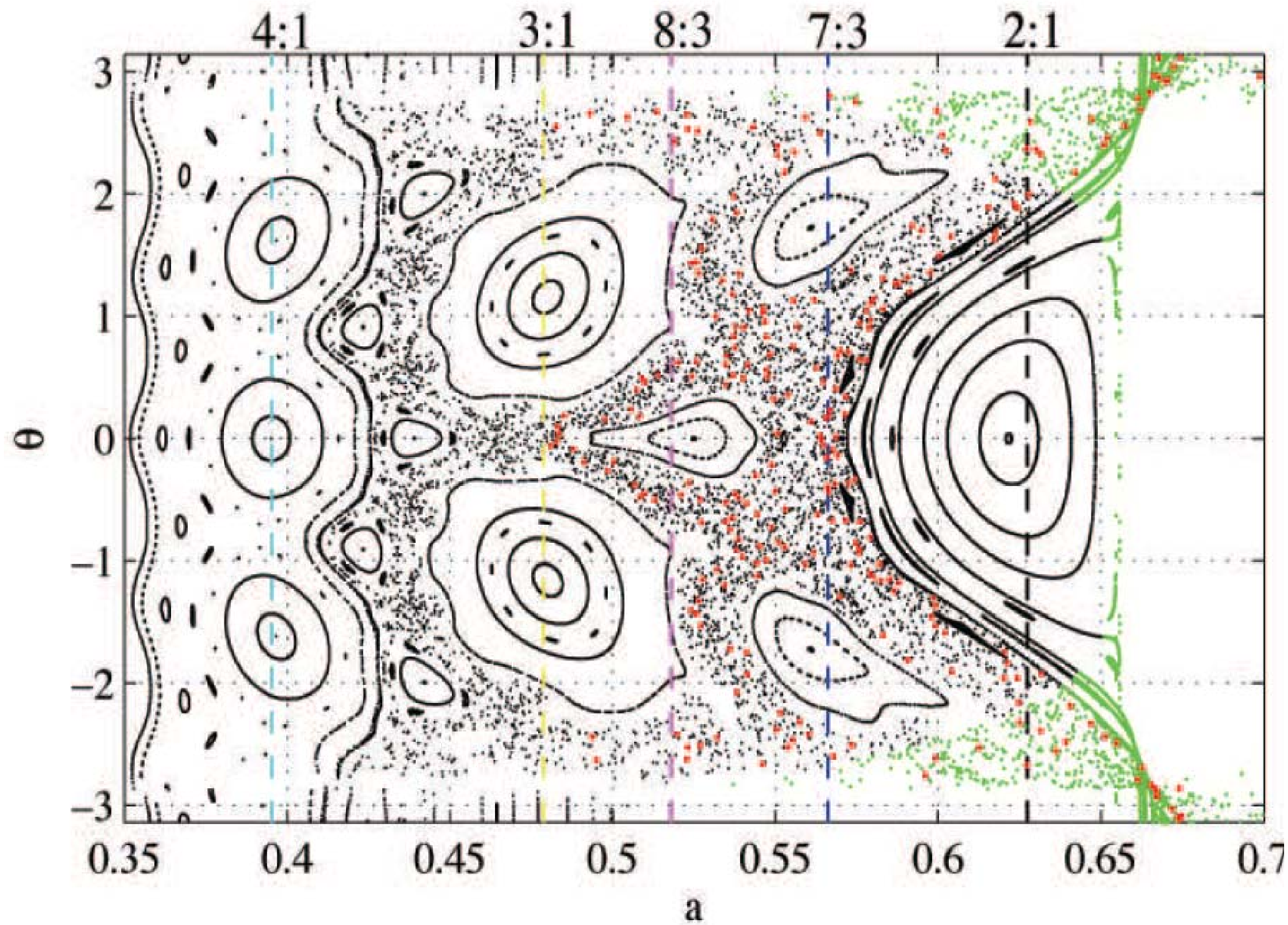
(d) $C = C(0.2)$.

(e) $C = C(0.3)$

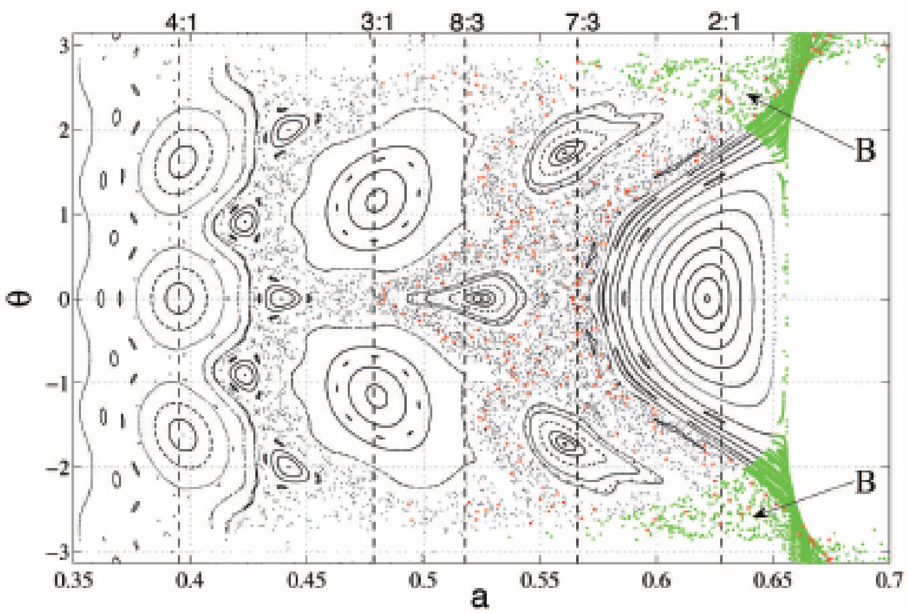
(f) $C = C(0.4)$.



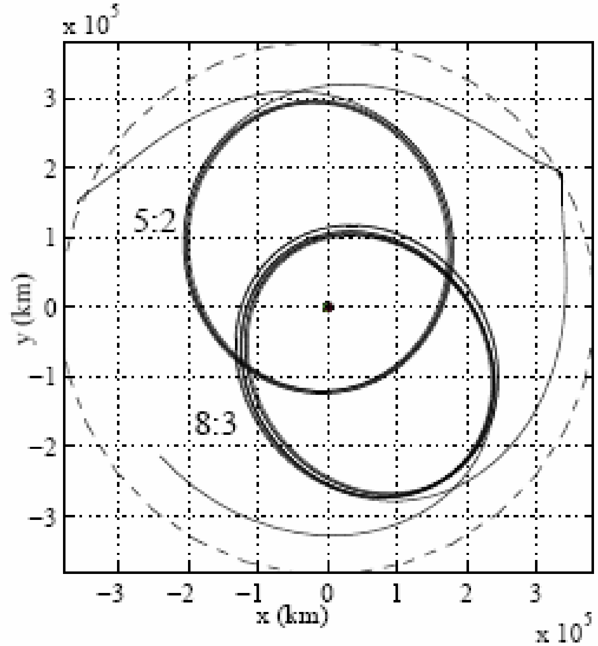
C =2.9734250513



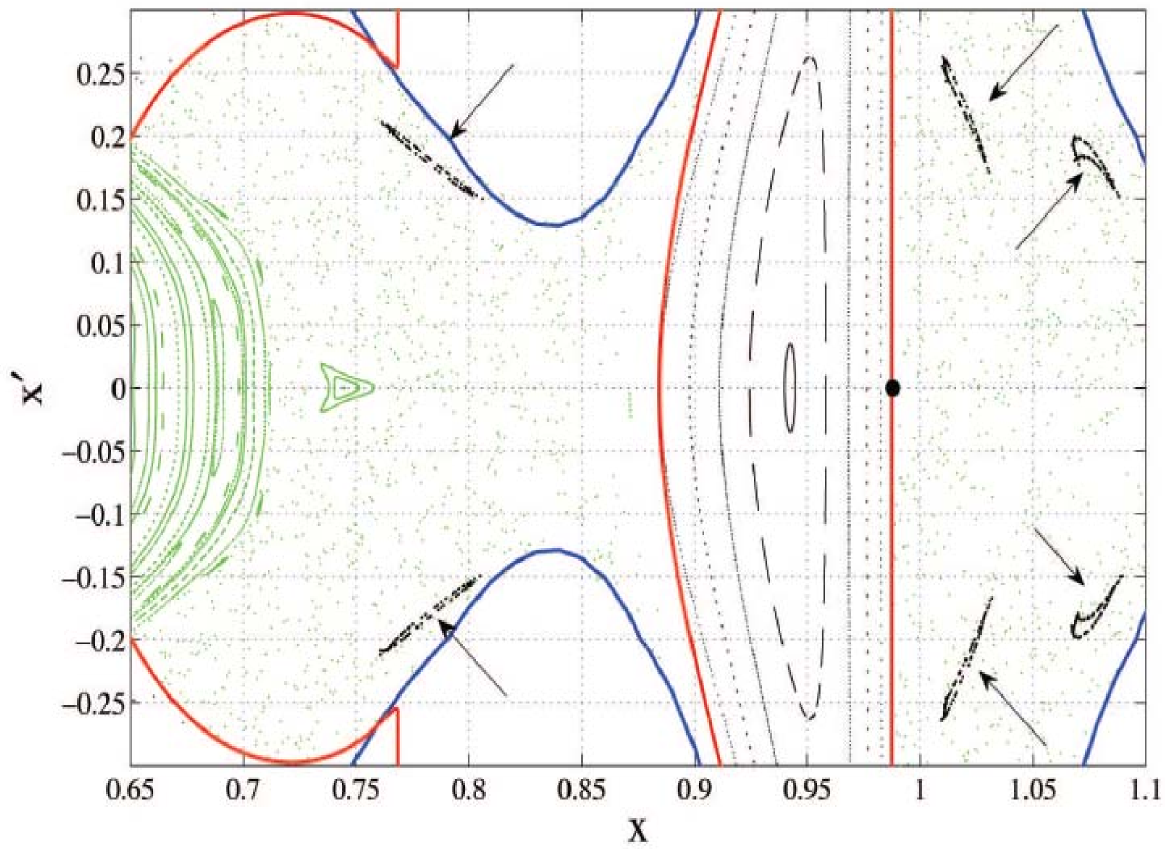
8:3 -> 5:2 Green - Extended WSB



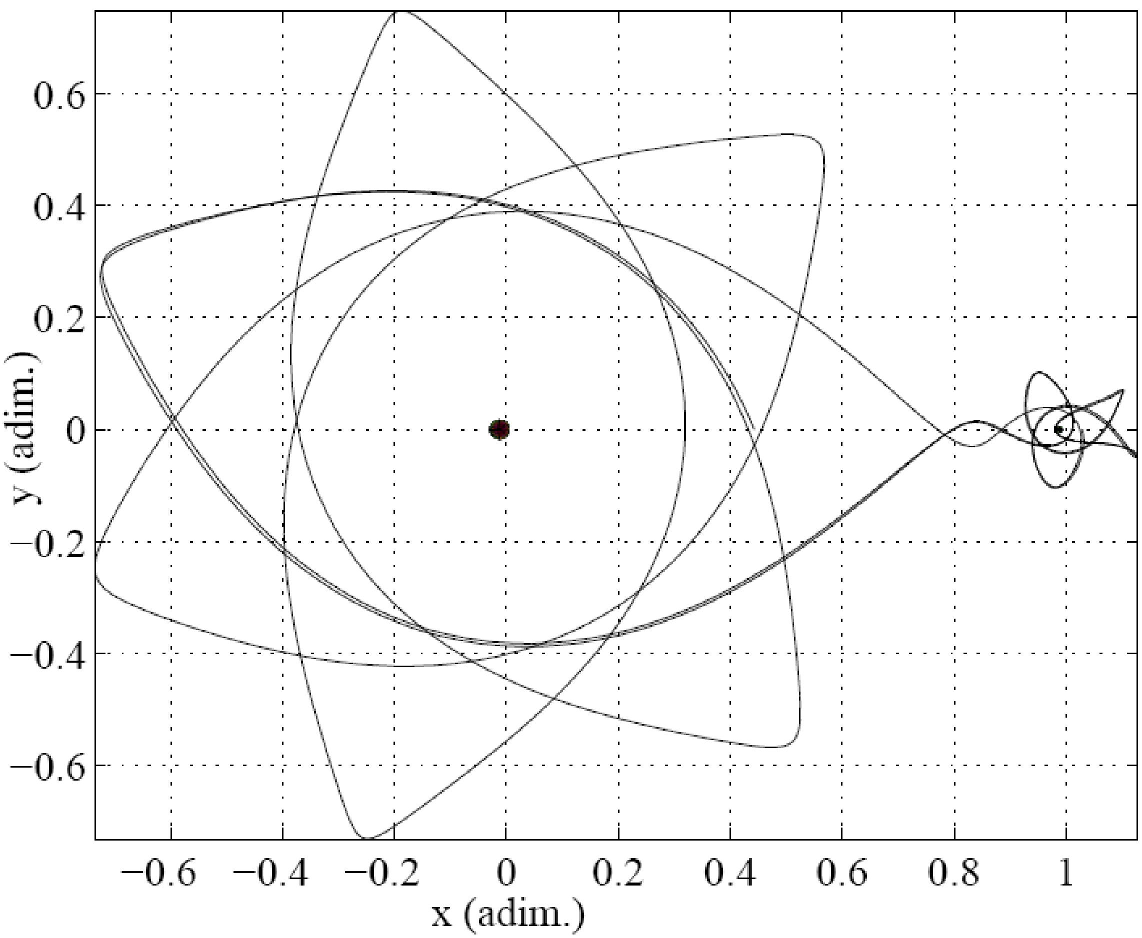
(a) (a, θ) plane.



(b) Inertial frame.



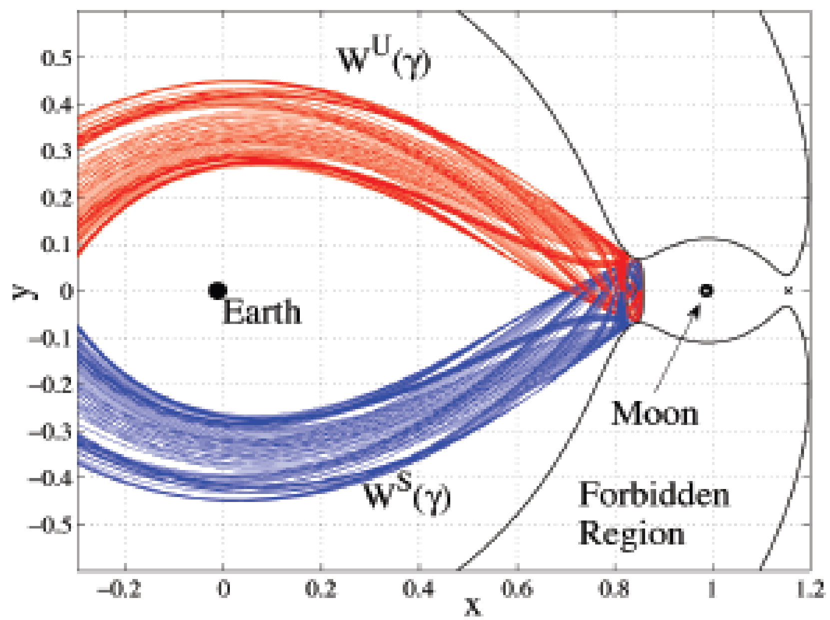
Interesting fractal Cantor regions identified with intriguing resonance transitions



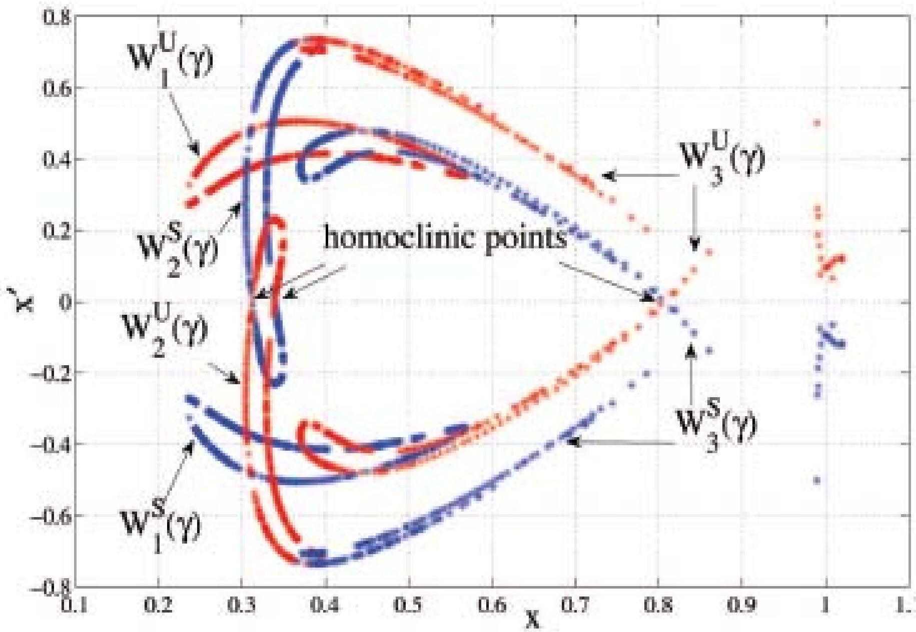
7:3 -> 7:3

Relationship to Lyapunov Orbits

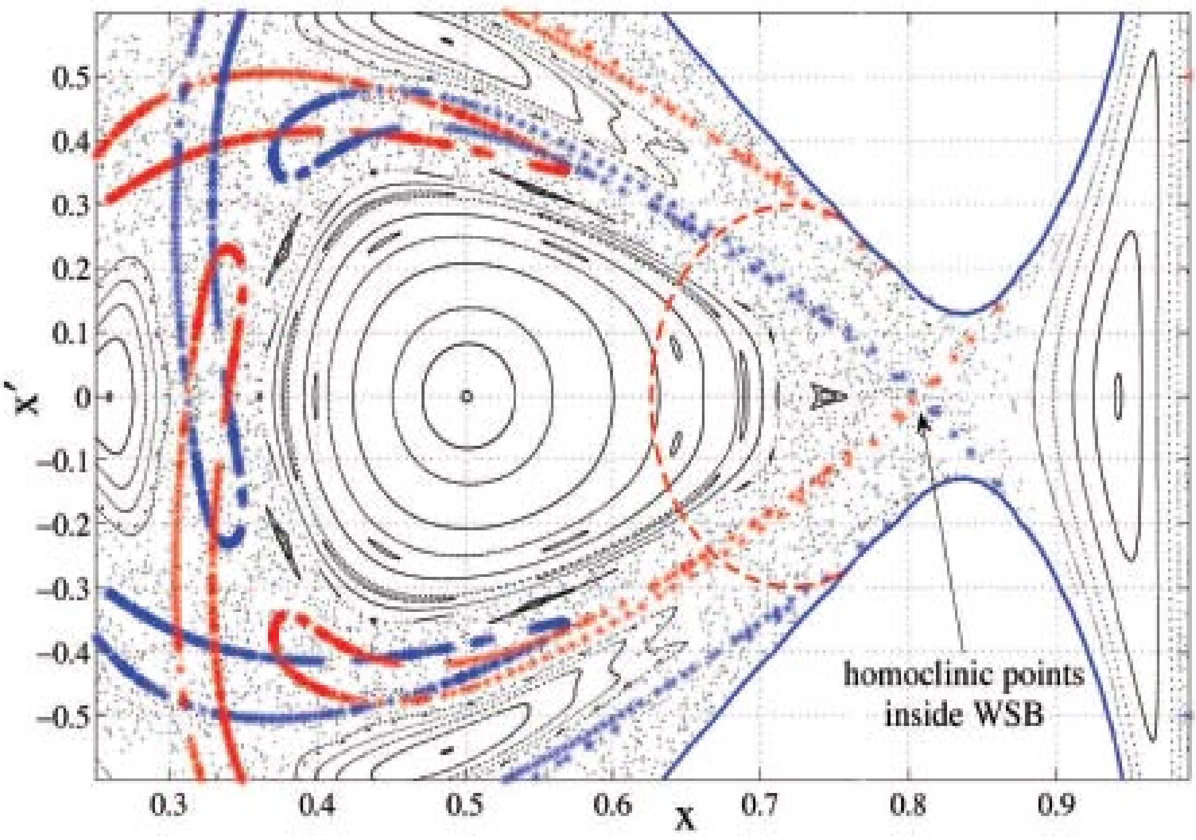
- The stable and unstable manifolds associated to the Lyapunov orbits intersect in a very complicated fashion – giving rise to chaotic motion.
- The chaotic regions fall within the WSB, which contains a key unstable feature – a homoclinic point.



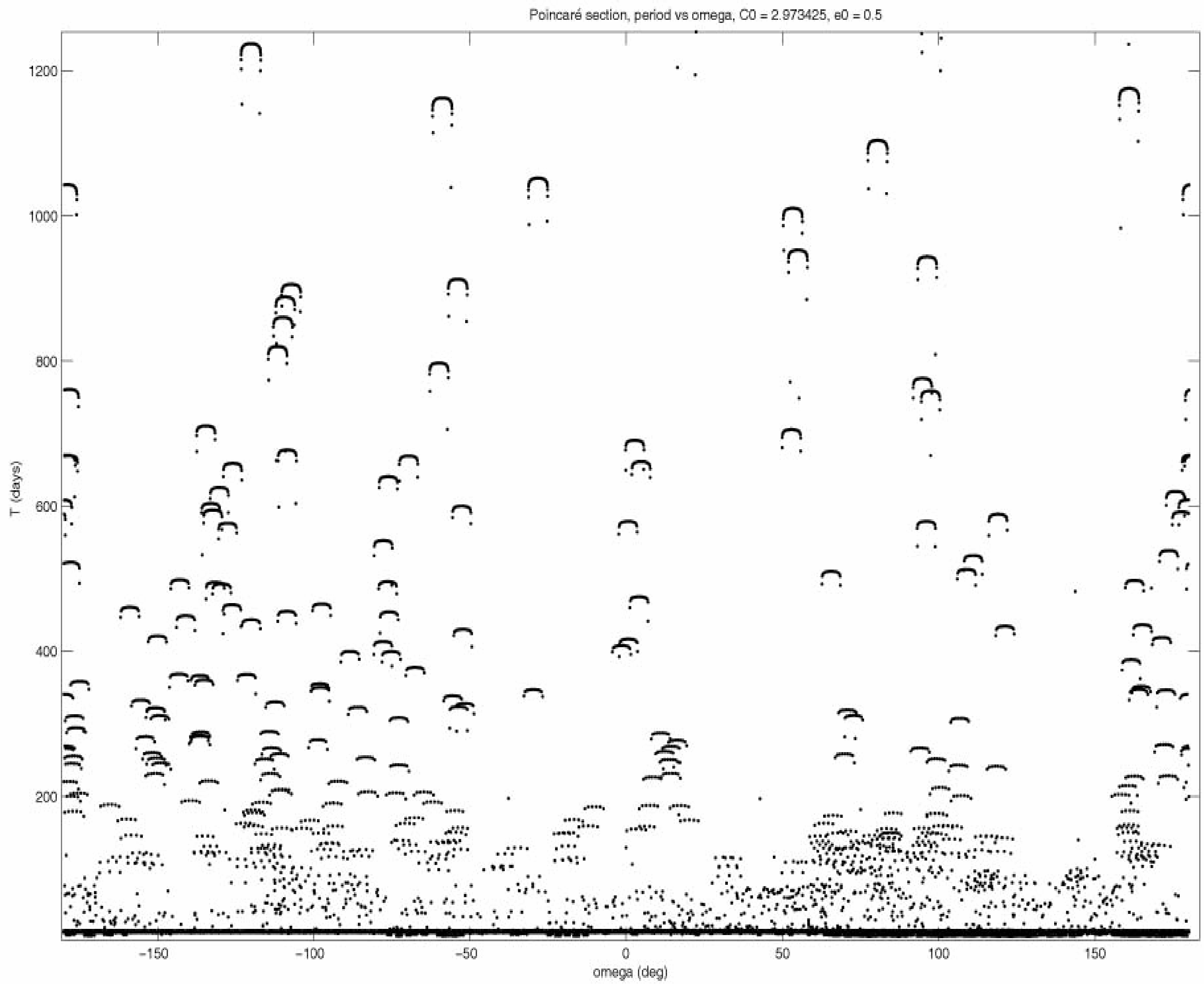
(a) The interior branch of the stable and unstable manifolds, $W^S(\gamma)$ and $W^U(\gamma)$, respectively, associated to a Lyapunov orbit about the L_2 point. The energy level is $C_0 = 3.18176\,83176$.



(b) Poincaré section of the two manifolds in figure 23(a) on the surface of section $\mathcal{S}$. Two transverse homoclinic points arise in the second intersection; another homoclinic point is defined by the third intersection.



0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
x′
0.3
0.4
0.5
0.6
0.7
0.8
0.9
x
homoclinic points
inside WSB



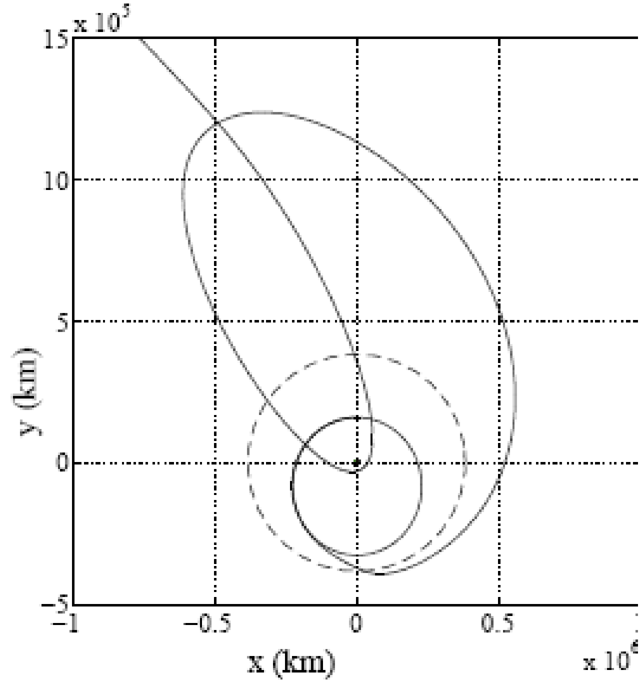
Poincaré section, period vs omega, C0 = 2.973425, e0 = 0.5
T (days)
omega (deg)
1200
1000
800
600
400
200
−150
−100
−50
0
50
100
150

Resonance States

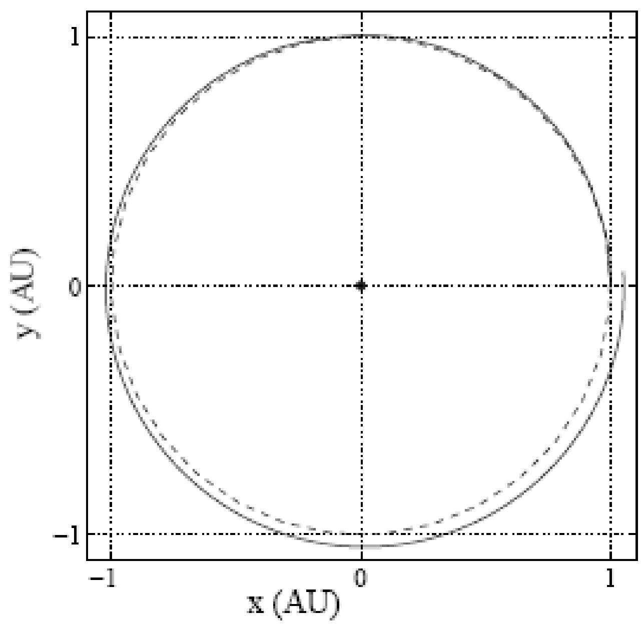
- Periods break up into discrete resonance states
- Analogous to quantum energy states of an atom

Applications

- Ballistic capture transfers are connecting resonance transitions
- Causing ejection from E-M system from weak capture yields a resonance orbit about the Sun in resonance with the Earth
- Obtain low energy escape transfers from E-M system(save 1 km/s!) Very promising
- ***Save substantial DV for Mars missions and beyond***



(a) Earth-centered frame.



(b) Sun-centered frame.

Ballistic Escape

- Way to ballistically escape E-M system
- Zero Delta V
- Mechanism:

  Resonance orbit about Earth in resonance with the Moon

  Interaction with lunar WSB

  Escape onto resonance orbit about the Sun, in resonance with the Earth

Connecting Resonance Orbits

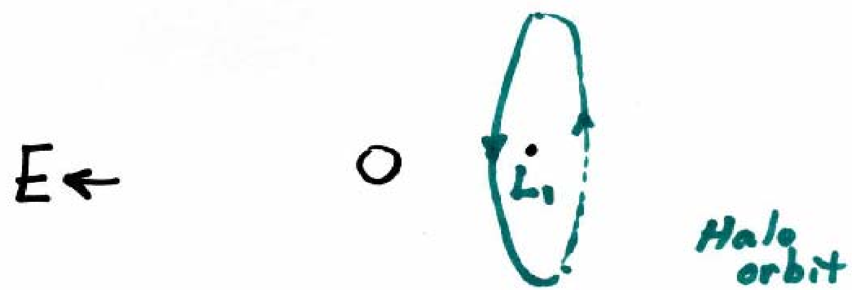
- Ballistic capture lunar transfer viewed as a trajectory interconnecting resonances in E-M system

- Ballistic escape from E-M system viewed as interconnecting resonance orbits about the Earth with those about the Sun

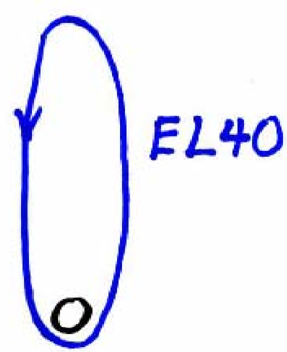
Applications, cont

- Can move in WSB about Moon for substantially less DV than by using conventional orbits – can reduce DV for inclination changes by a factor of ***12***
- No need for using halo orbits about lunar L1-L2 points for a comm system
- Using new low energy WSB orbits more effective – *implications on lunar architecture, together with WSB transfer*

Replace



With



Future Work

- New results on WSB make large opening with many intriguing results waiting to be uncovered
- Potentially new types of motions
- Transfer of material between planetary systems (with Amaya Moro-Martin and Renu Maholtra) - Lithopanspermia Hypothesis true?

Acknowledgement

Research funded by NASA SMD/AISR

References

- E. Belbruno, Mission Extension Using Sensitive Trajectories and Autonomous Control, NASA AISRP Annual Report(3/06-3/07), #NASA-2-ARPT-07, March 1, 2007
- A New Class of Low Energy Lunar Orbits with Applications, Proceedings of New Trends in Astrodynamics and Applications, American Institute of Physics, Feb 2007.

- Resonance Transitions Associated to Weak Capture in the Restricted three-Body Problem, Submitted for Publication to *Chaos* June 2007 (with F. Topputo, M. Gidea) www.edbelbruno.com/research/ResonanceTransition.pdf

(Details of the results of this presentation)

Resonant Motion, Ballistic Escape, and their Application to Astrodynamics, Submitted for publication to AIP – September 2007

Books

- *Capture Dynamics and Chaotic Motions in Celestial Mechanics(With the Construction of Low Energy Transfers)*, Princeton University Press, 2004
- *Fly Me to the Moon: An Insiders Guide to the New Science of Space Travel,* Princeton University Press, March 2007